The analysis of streamflow prediction uncertainty of CREW model using GLUE

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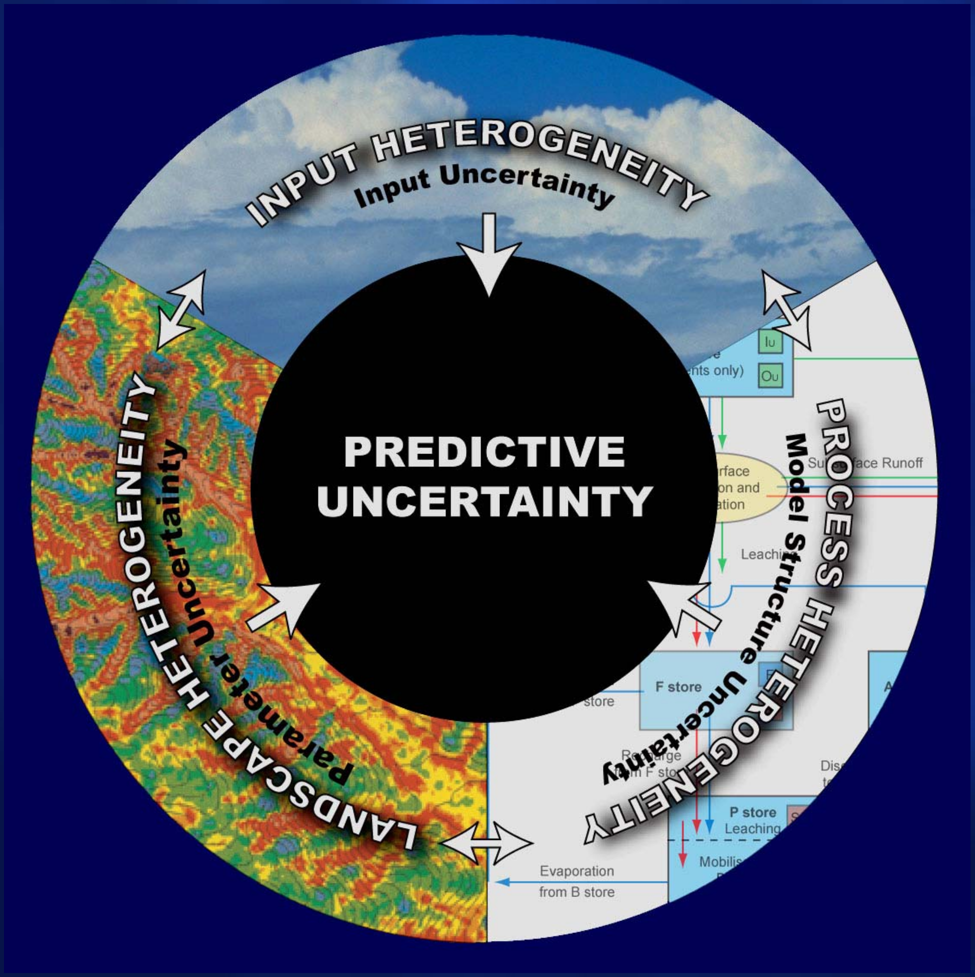
Hydrologic model prediction uncertainty ?

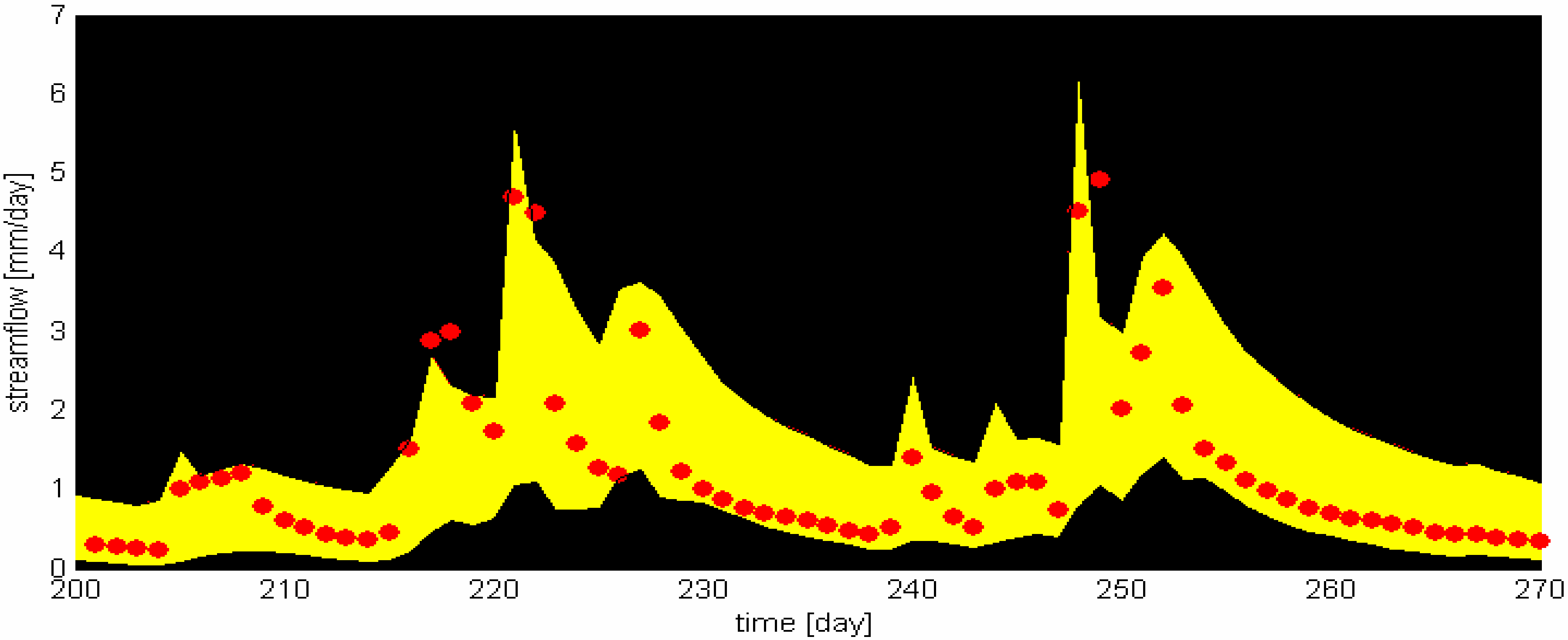


Predictive uncertainty =

Input uncertainty
+
Parameter uncertainty
+
Model structure uncertainty
+
Initial condition uncertainty

Predictive uncertainty and links with climatic and landscape heterogeneity (Sivapalan et al., 2003)

Streamflow prediction uncertainty



How ?
- ➢ Construct prediction bands
- ➢ Monte Carlo Simulation

Why ?
- ➢ To assess overall model prediction ability
- ➢ To learn the way to reduce prediction uncertainty

Outline

1. CREW: a physically based and distributed hydrological model at the catchment scale

2. Study area: Howard springs and Susannah brook

3. The analysis of model uncertainty in streamflow prediction
   - 3.1 construction of uncertainty bands in streamflow prediction
   - 3.2 the value of additional data: uncertainty quantification and reduction

1. CREW

A physically based and distributed hydrological model at the catchment scale

CREW (Lee et al., 2006)

- Balance of mass and momentum at the scale of catchment: Reggiani (1998, 1999)
- Benefits: physically sound,
  less input data requirement,
  less computational cost,
  suitable for large scale modeling (~ > 100 km^2)
- Application: Weiherbach, Germany (Lee et al., 2006)
  Collie river basin, Australia (Lee et al., 2006)

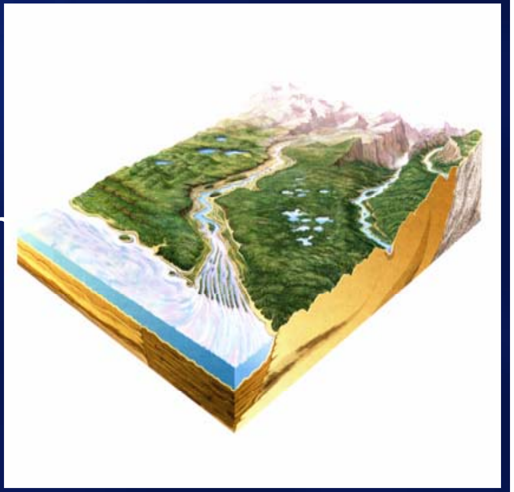
Reggiani *et al.*'s theory

Spatial scale

Basin-scale theory and modeling

Darcy
Richards
(Micro-scale)

Mass & momentum
balance
(Basin scale)



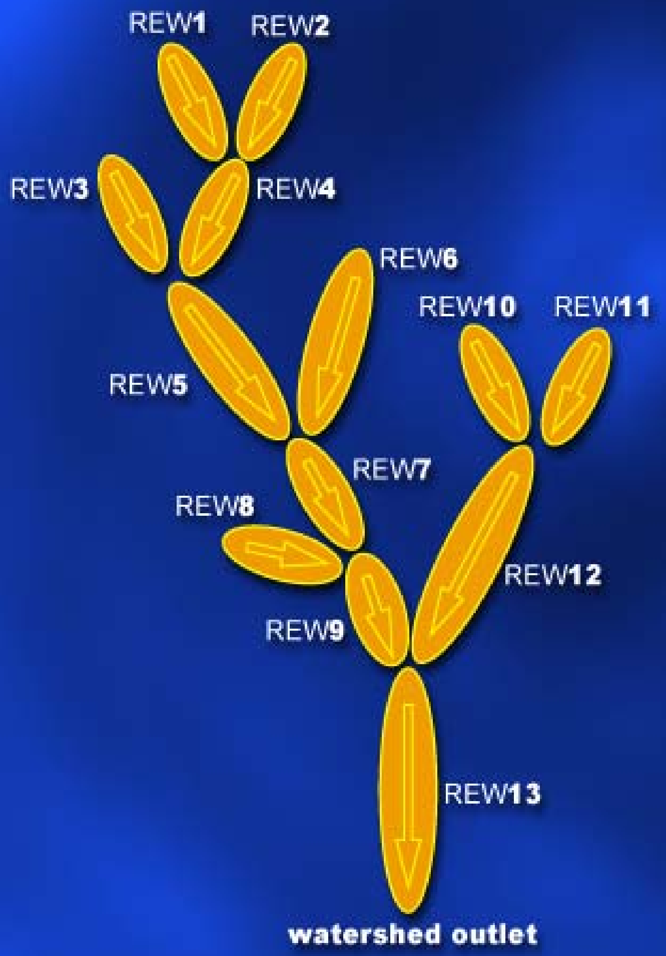
Micro-scale theory and modeling

Freeze & Harlan (1969)

Reggiani *et al.* (1998,1999,2000)

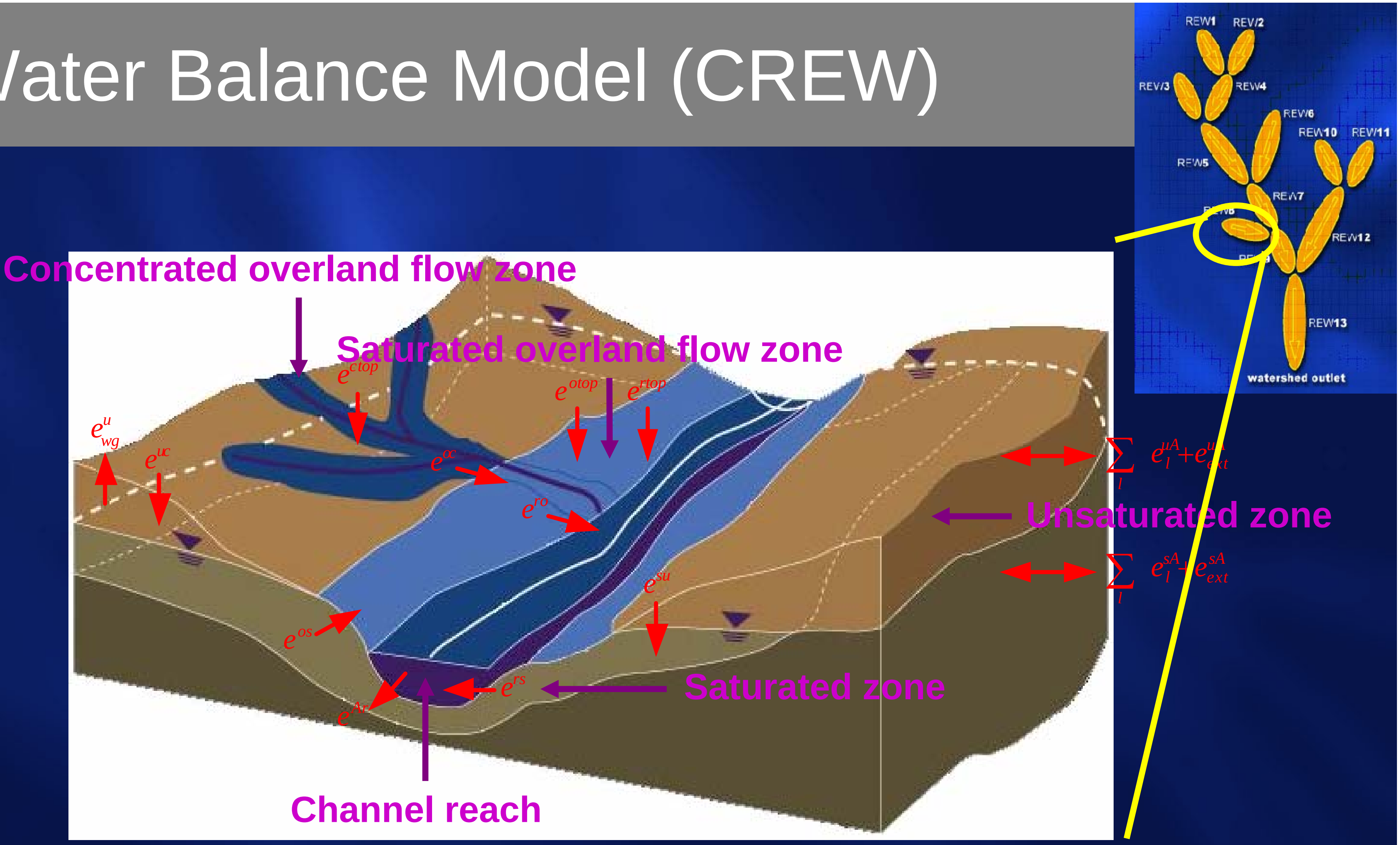
- http://www.fsl.orst.edu/lter/research/component/hydro/summary.cfm?sum=dye02&topnav=62
- Yoshi (2003)

Discretization: 1 catchment ➔ 13 analysis units



Reggiani *et al*. (1998)

Water Balance Model (CREW)



e^{ij}: exchange mass fluxes between i and j zone

Reggiani et al (1999)

Governing equations (CREW)

- ***Mass balance equations*** (Reggiani et al., 1998, 1999; Lee et al., 2006)

Unsaturated Zone (U)	$\underbrace{\varepsilon \frac{d}{dt}(y^u \omega^u s^u)}_{\text{storage}} = \underbrace{\min\left[i\omega^u, \overline{K_s} + \alpha^{uc} \frac{\overline{K}_s \lvert\Psi\rvert (1-s^u)\varepsilon}{s^u y^u}\right]}_{\text{infilteration}} - \underbrace{\varepsilon\omega^u v_z^u}_{\text{recharge or capillary rise}} - \underbrace{\min\left[e_p + Mk_v \overline{e}_p, \alpha^u_{wg} \frac{\overline{K}_s}{(1-s^u)y^u} \frac{(s^u)^{2+d}\varepsilon\lvert\Psi_b\rvert}{m}\right]}_{\text{evapotranspiration}}$
Saturated Zone (S)	$\underbrace{\varepsilon \frac{d}{dt}(y^s)}_{\text{storage}} = \underbrace{\varepsilon\omega^u v_z^u}_{\text{recharge or capillary rise}} \underbrace{- \omega^o \alpha_1^{os} \overline{K}_s^{\alpha_2^{os}} \left[\frac{y^u s^u \omega^u + y^s}{Z\lvert\Psi\rvert}\right]^{\alpha_3^{os}}}_{\text{seepage}} - \underbrace{q_s}_{\text{sat. zone-river exchange}}$
Concentrated Overland Flow Zone (C)	$\underbrace{\frac{d}{dt}(y^c \omega^c)}_{\text{storage}} = \underbrace{\omega^c J}_{\text{rainfall or evaporation}} - \underbrace{\min\left[i\omega^u, \quad \overline{K_s} + \alpha^{uc} \frac{\overline{K}_s \lvert\Psi\rvert (1-s^u)\varepsilon}{s^u y^u}\right]}_{\text{infilteration}} - \underbrace{\alpha^{oc}\xi^r y^c v^c}_{\text{flow to saturated overland flow zone}}$
Saturated Overland Flow Zone (O)	$\underbrace{\frac{d}{dt}(y^o \omega^o)}_{\text{storage}} = \underbrace{\omega^o \alpha_1^{os} \overline{K}_s^{\alpha_2^{os}} \left[\frac{y^u s^u \omega^u + y^s}{Z\lvert\Psi\rvert}\right]^{\alpha_3^{os}}}_{\text{seepage}} + \underbrace{\alpha^{oc}\xi^r y^c v^c}_{\text{inflow from conc. overl. flow}} + \underbrace{\omega^o J}_{\text{rainfall or evaporation}} - \underbrace{\alpha^{ro}\xi^r y^o v^o}_{\text{lateral channel inflow}}$
Channel Reach Zone (R)	$\underbrace{\frac{d}{dt}(m^r \xi^r)}_{\text{storage}} = \underbrace{\alpha^{ro}\xi^r y^o v^o}_{\text{lateral channel inflow}} + \underbrace{q_s}_{\text{sat. zone-river exchange}} + \underbrace{\sum_l \frac{m_l^r v_l^r}{\Sigma}}_{\text{inflow}} - \underbrace{\frac{m^r v^r}{\Sigma}}_{\text{outflow}} + \underbrace{\xi^r w^r J}_{\text{rainfall, evaporation on free surface}}$

Governing equations (CREW)

- ***Momentum balance equations*** (Reggiani et al., 1998, 1999, 2000)

Unsaturated Zone (U)	$v_z^u = \overline{K}\left[-s^u + \frac{1}{2} + \frac{\lvert\Psi\rvert}{y^u}\right]$	The catchment scale Darcy's law
Saturated Zone (S)	ignoring	
Concentrated Overland Flow Zone (C)	$v^c = \frac{1}{n_m^c}\left[y^c\right]^{2/3}\left[\sin(\gamma^c)\right]^{1/2}$	The catchment scale Manning eq.
Saturated Overland Flow Zone (O)	$v^o = \frac{1}{n_m^o}\left[y^o\right]^{2/3}\left[\sin(\gamma^o)\right]^{1/2}$	The catchment scale Manning eq.
Channel Reach Zone (R)	$v^r = \frac{1}{n_m^r}\sqrt{\frac{\left[R^r\right]^{1/3}}{P^r l^r}\left[m^r l^r \sin(\gamma^r) \pm \sum_l \{\frac{1}{4} y^r (m^r + m^l)\cos\delta_l\} - \frac{1}{2} y^r m^r\right]}$	

The catchment scale Diffusion wave eq.

2. Study area

Howard springs

Stan Skymanski (2006)

Susannah brook

Photo: Carlos Ocampo

Howard springs

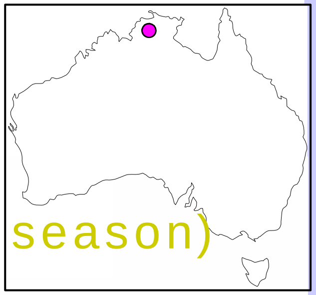
126 km^2

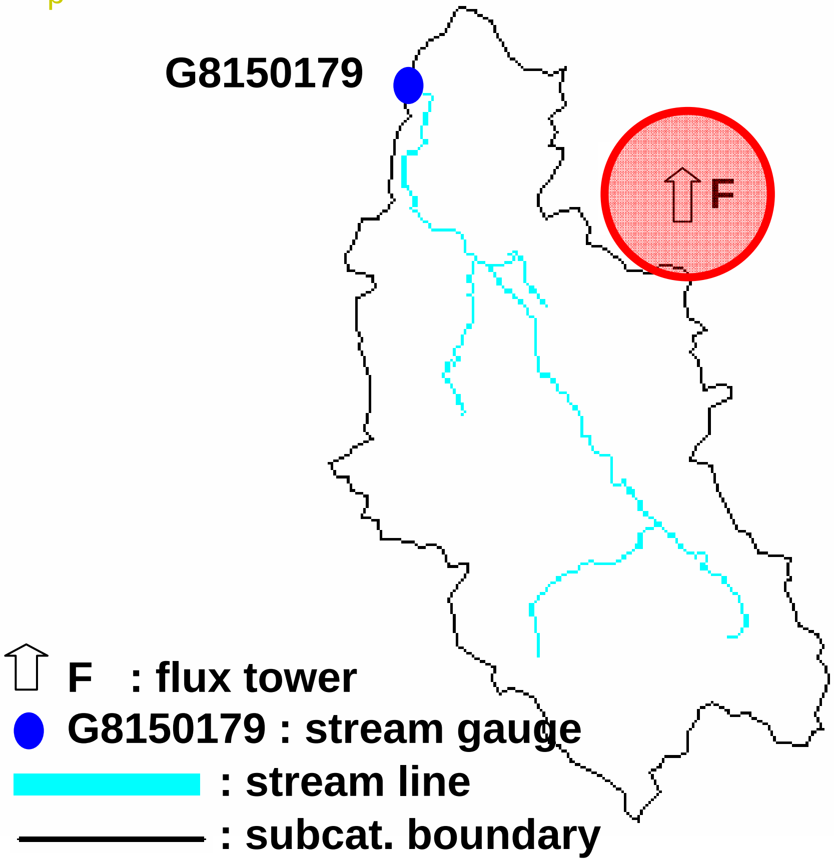
R=2256 [mm/yr]

E_p=2238 [mm/yr]

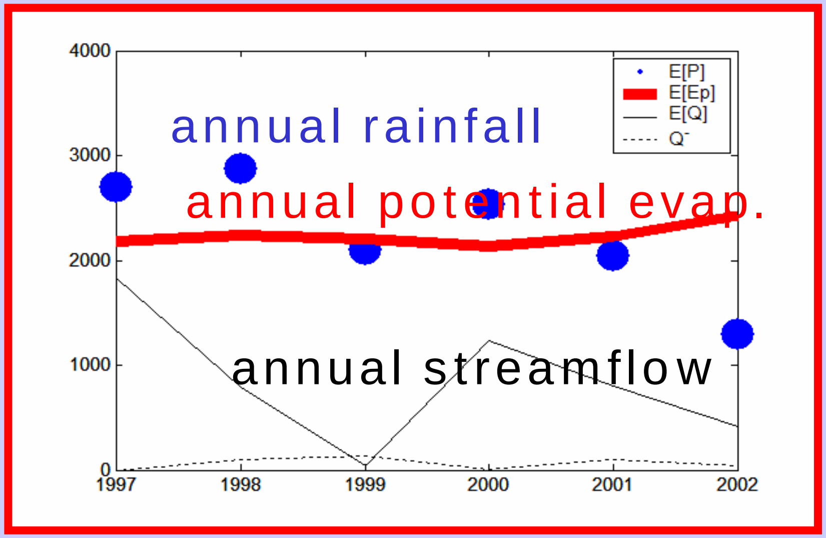
RC=Q/R=0.33-0.48 (wet season)

DI=E_p/R=0.99

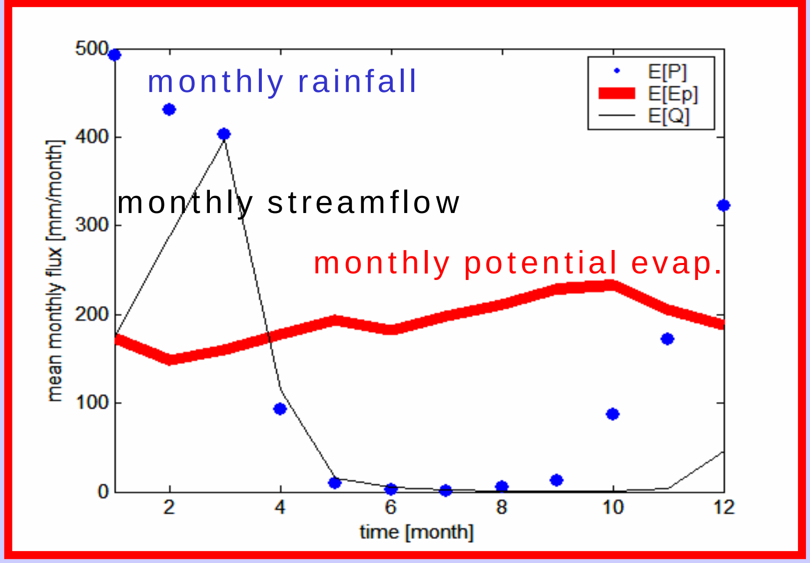




Annual flux

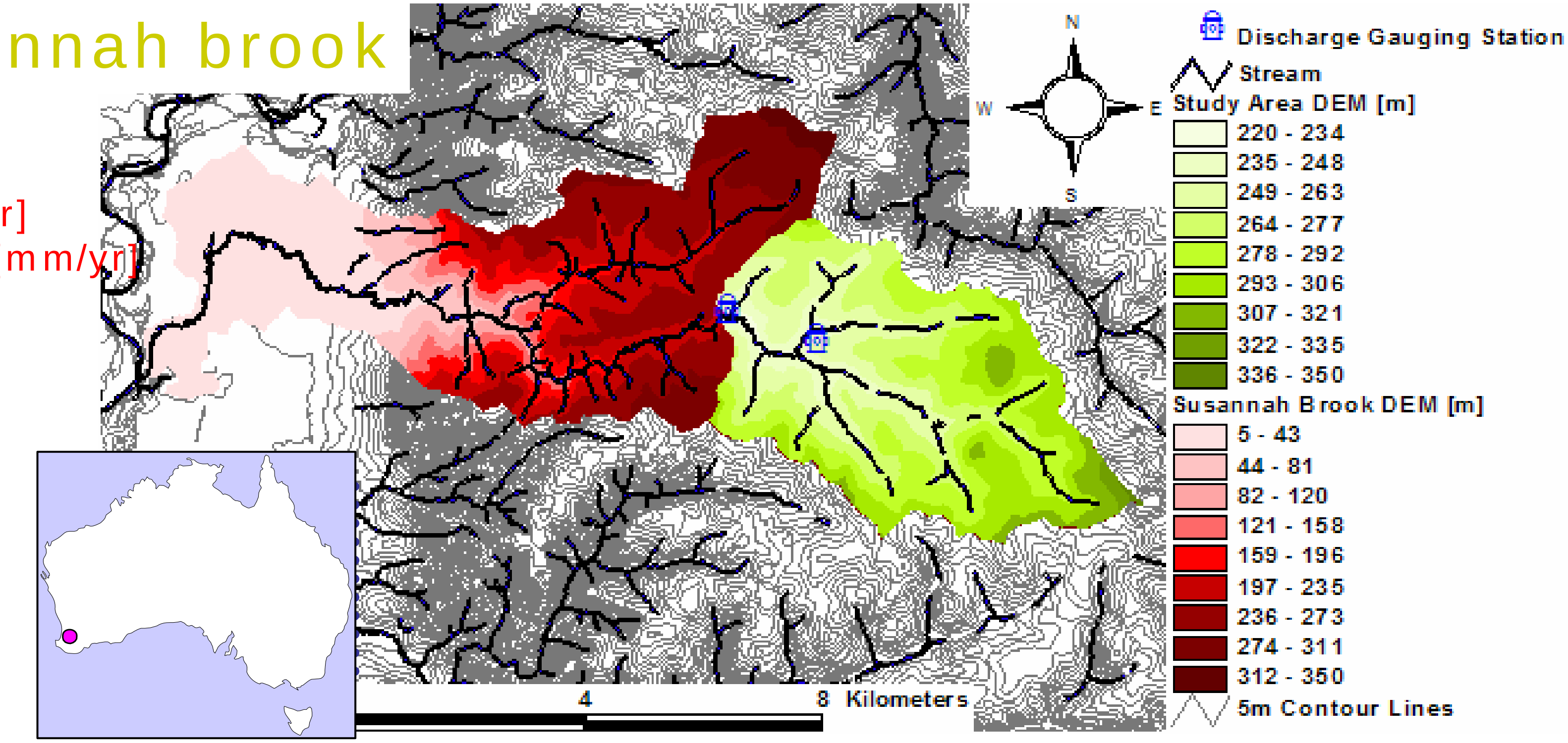


Mean monthly flux

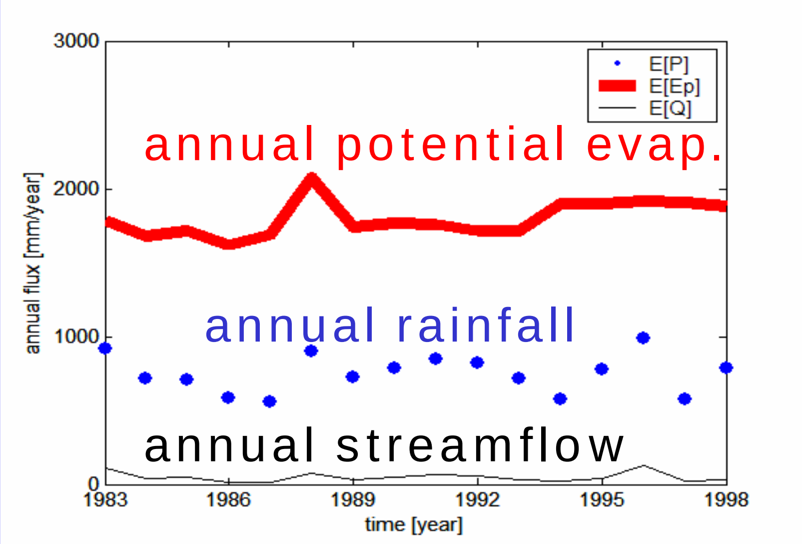


Susannah brook

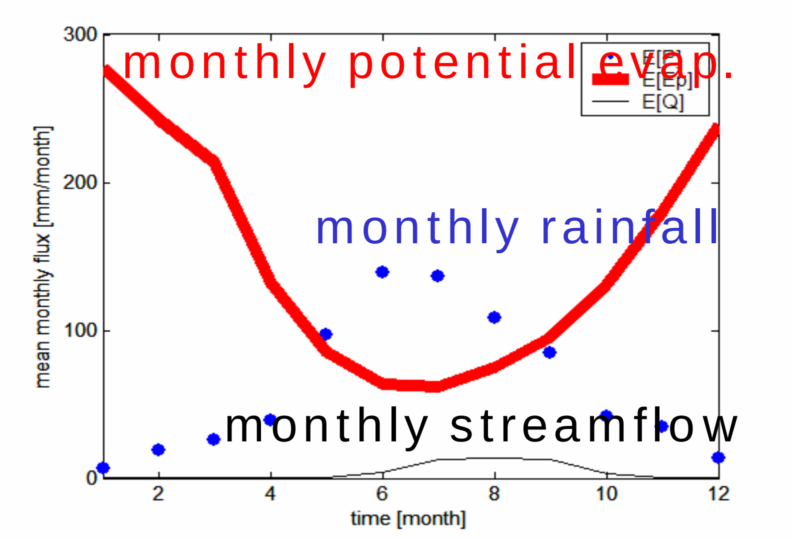
23 km^2
R=872 [mm/yr]
Pan E=2130 [mm/yr]
RC=Q/R=0.17
DI=E/R=2.5



Annual flux



Mean monthly flux



3. The analysis of model uncertainty in streamflow prediction

3.1. construction of uncertainty bands in streamflow prediction

Generalized Likelihood Uncertainty Estimation (GLUE; Beven and Binley, 1992)

- A Bayesian Monte-Carlo simulation-based technique

- Likelihood measure:

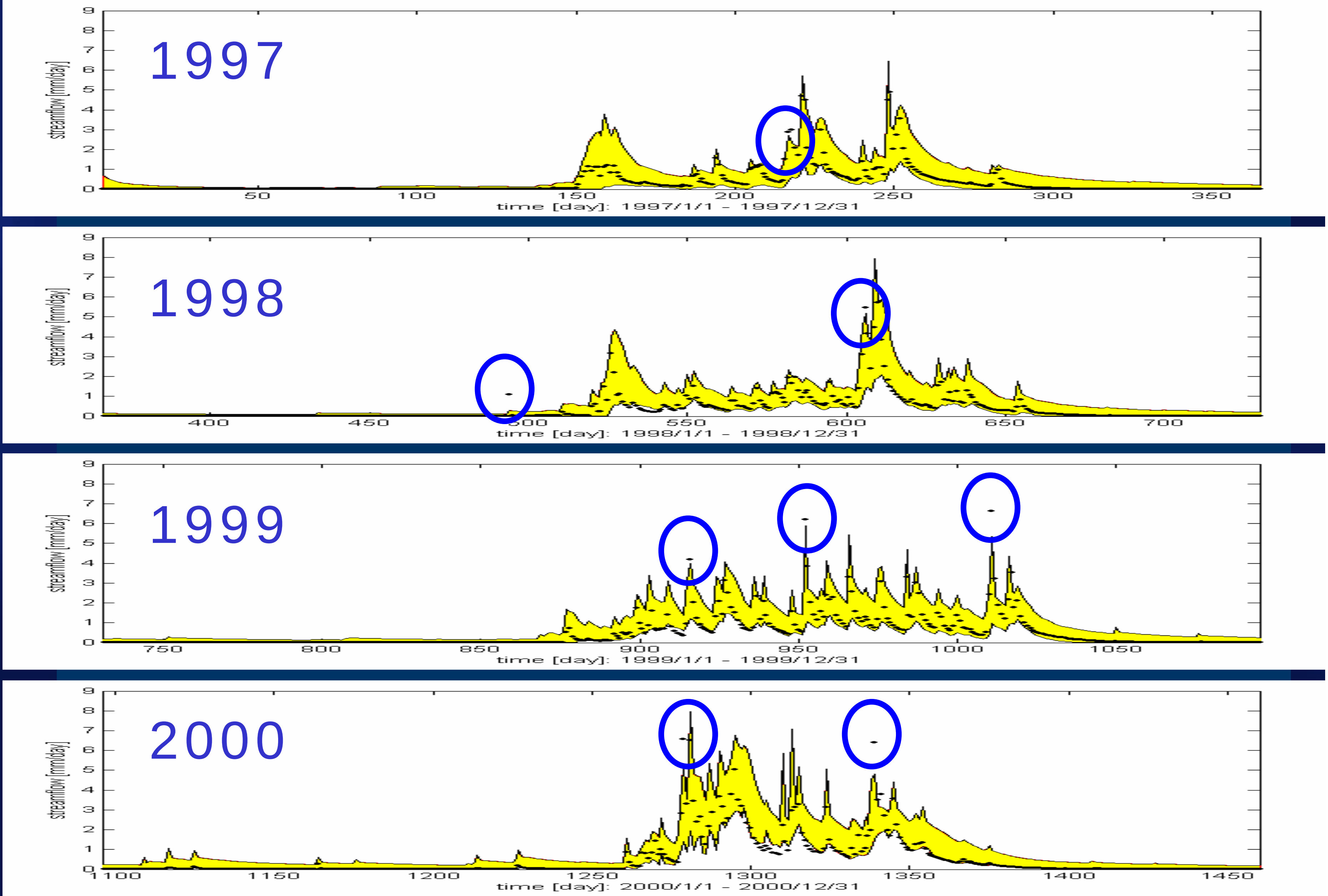
$$L(\theta_i | Y) = 1 - \frac{\sigma_i^2}{\sigma_o^2} \qquad (\sigma_i^2 < \sigma_o^2)$$

θ_i : parameter set Y : data

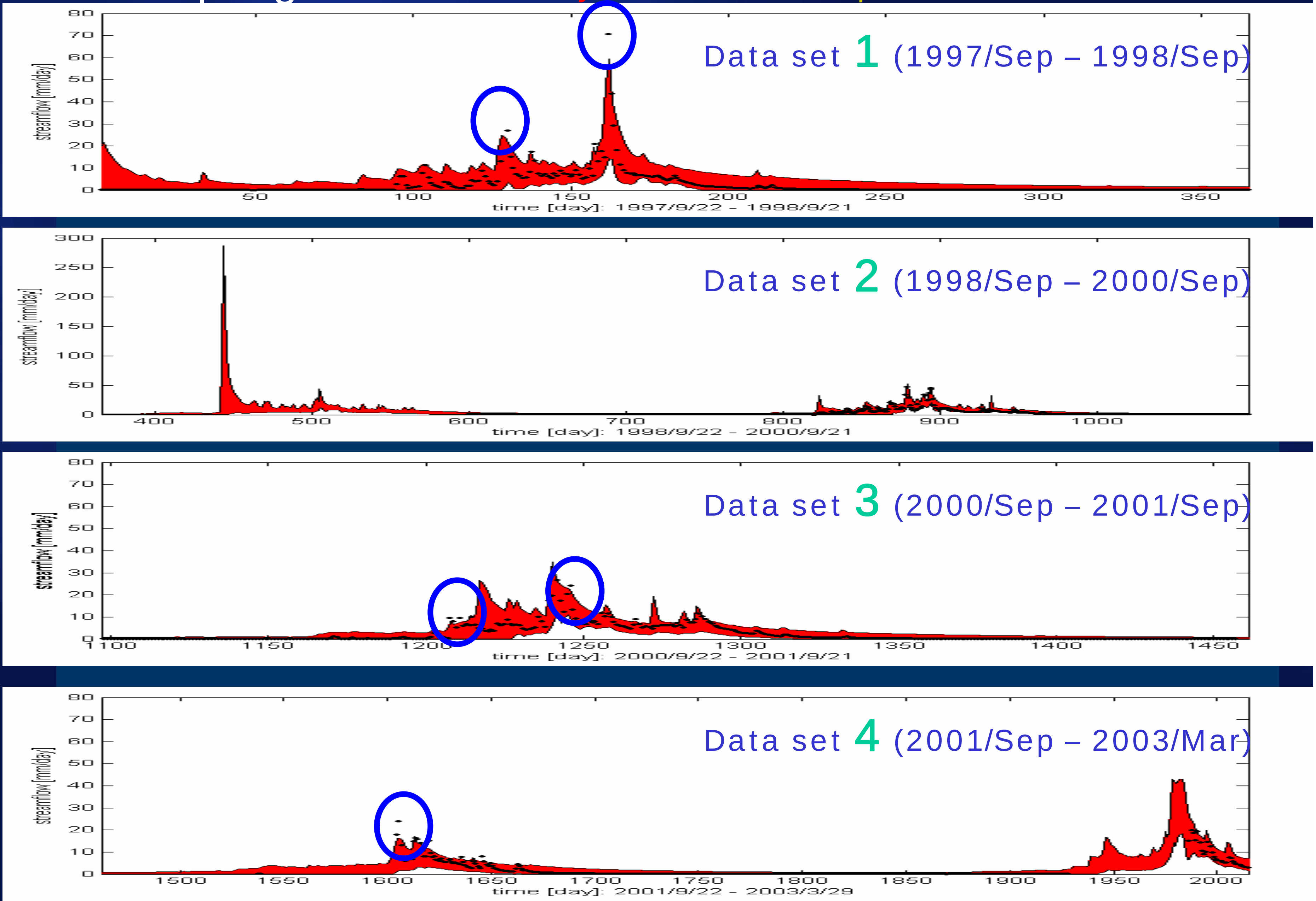
σ_i^2 : error variance σ_o^2 : observed variance

- Parameter sets: 30,000

Susannah brook: 99% Uncertainty bounds in runoff prediction



Howard springs: 99% Uncertainty bounds in runoff prediction

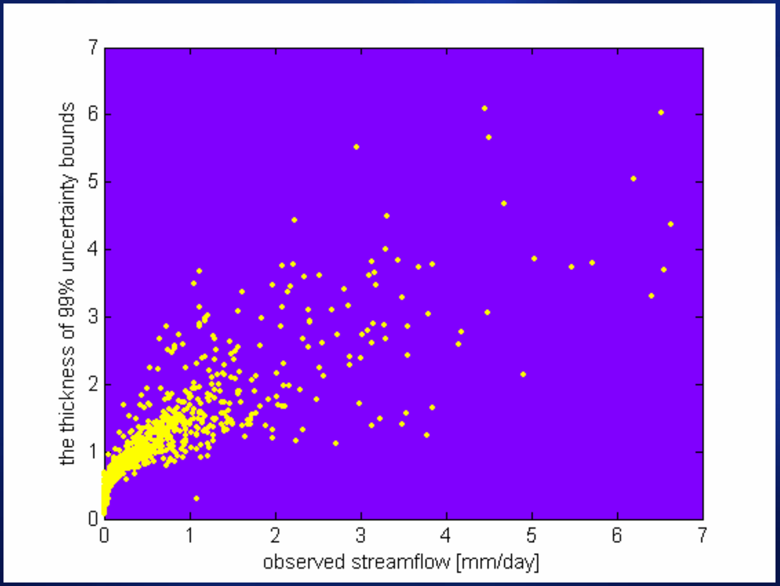


Prediction uncertainty in streamflow

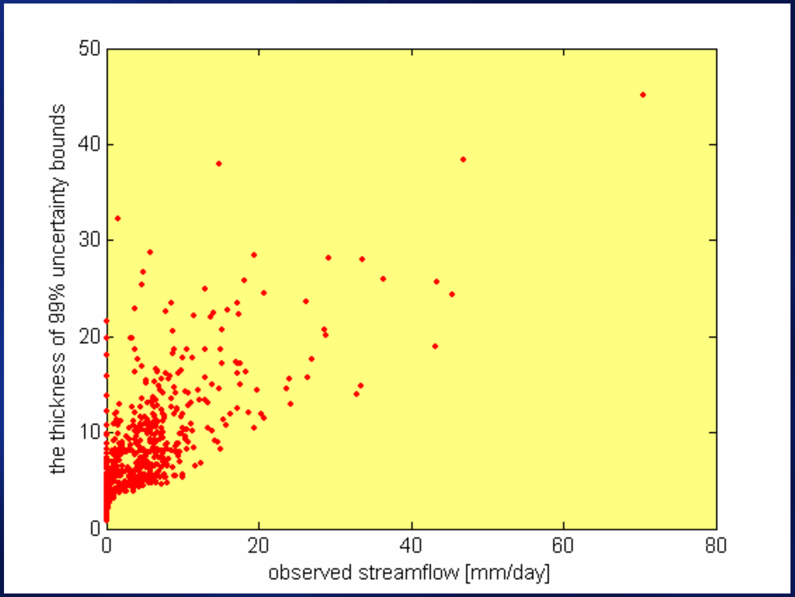
Susannah brook

Howard springs

Thickness of u-bounds

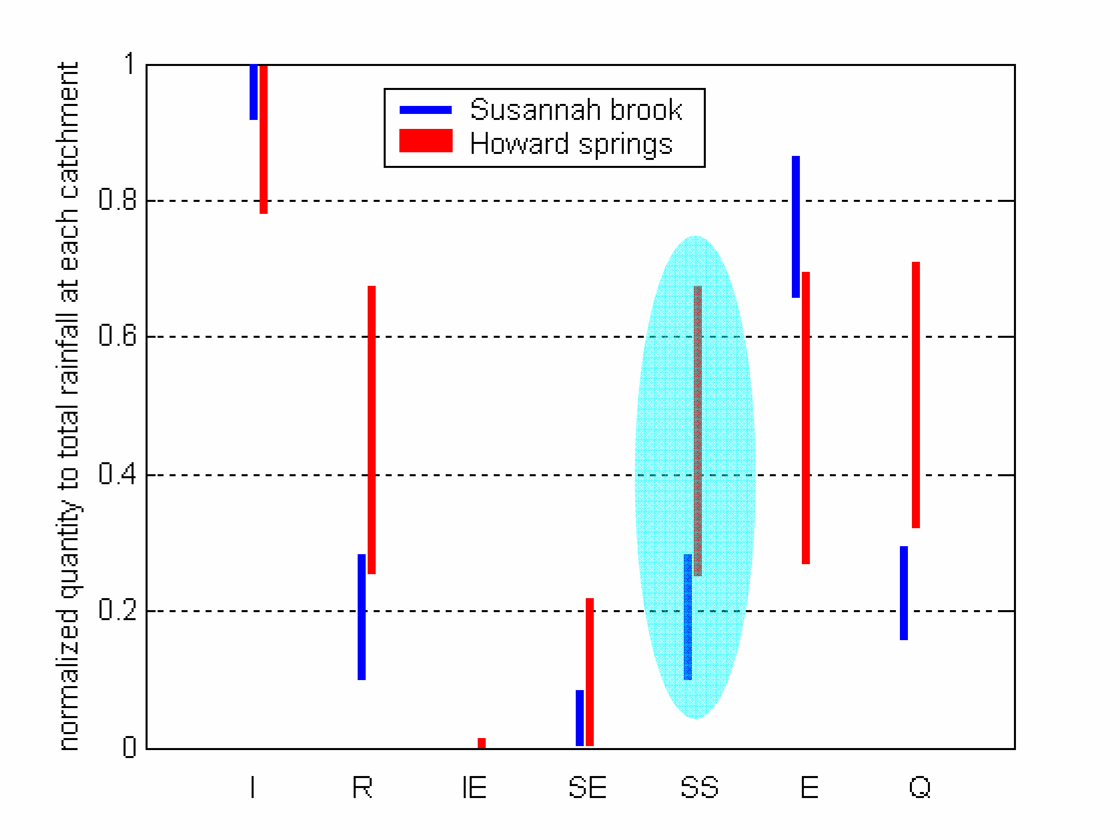


Observed streamflow



Observed streamflow

Prediction uncertainty in annual water balance



I : infiltration

R : recharge

IE : infilteration excess

SE: saturation excess

SS: subsurface flow

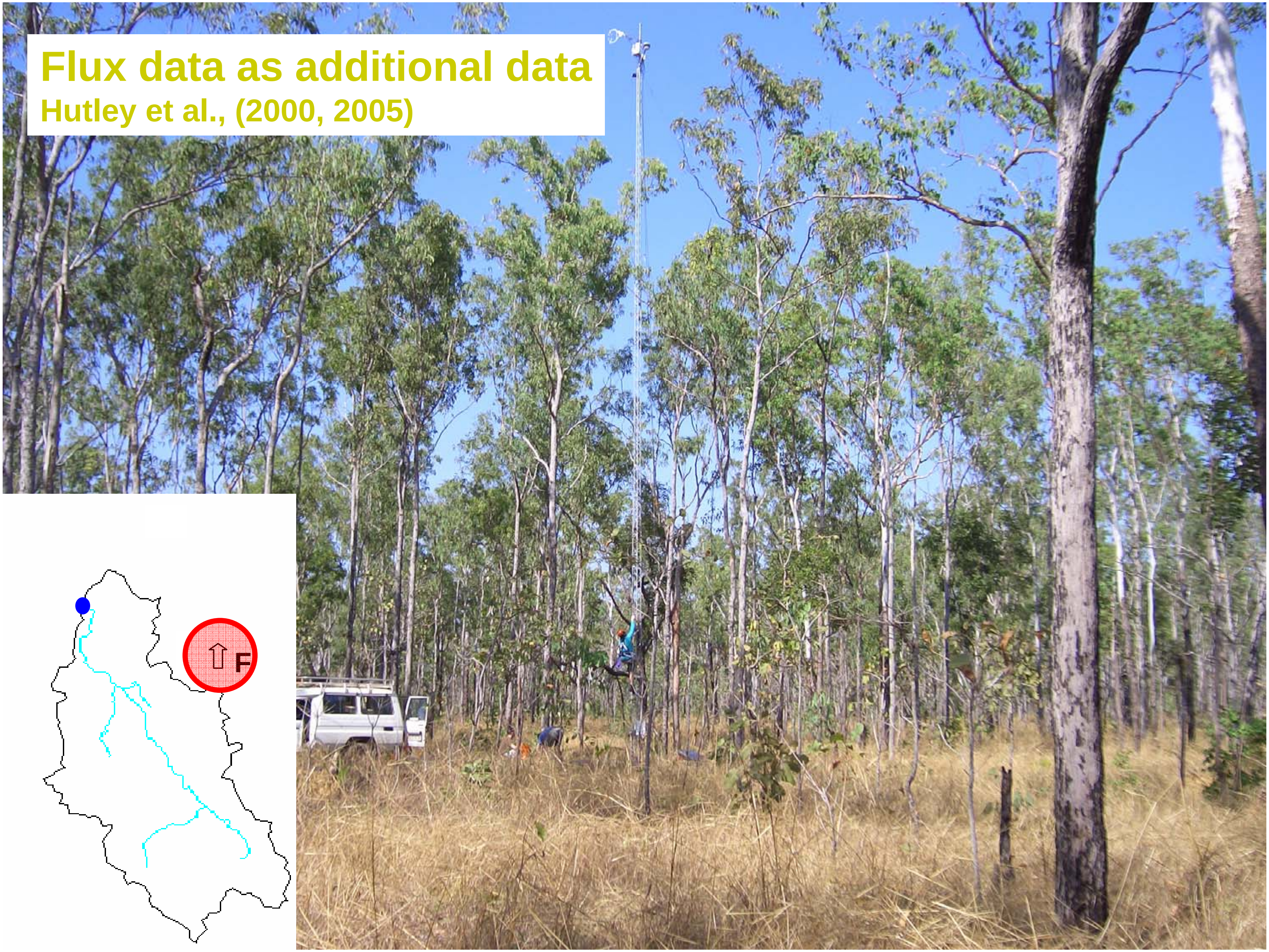
E : simulated evaporation

Q : simulated streamflow

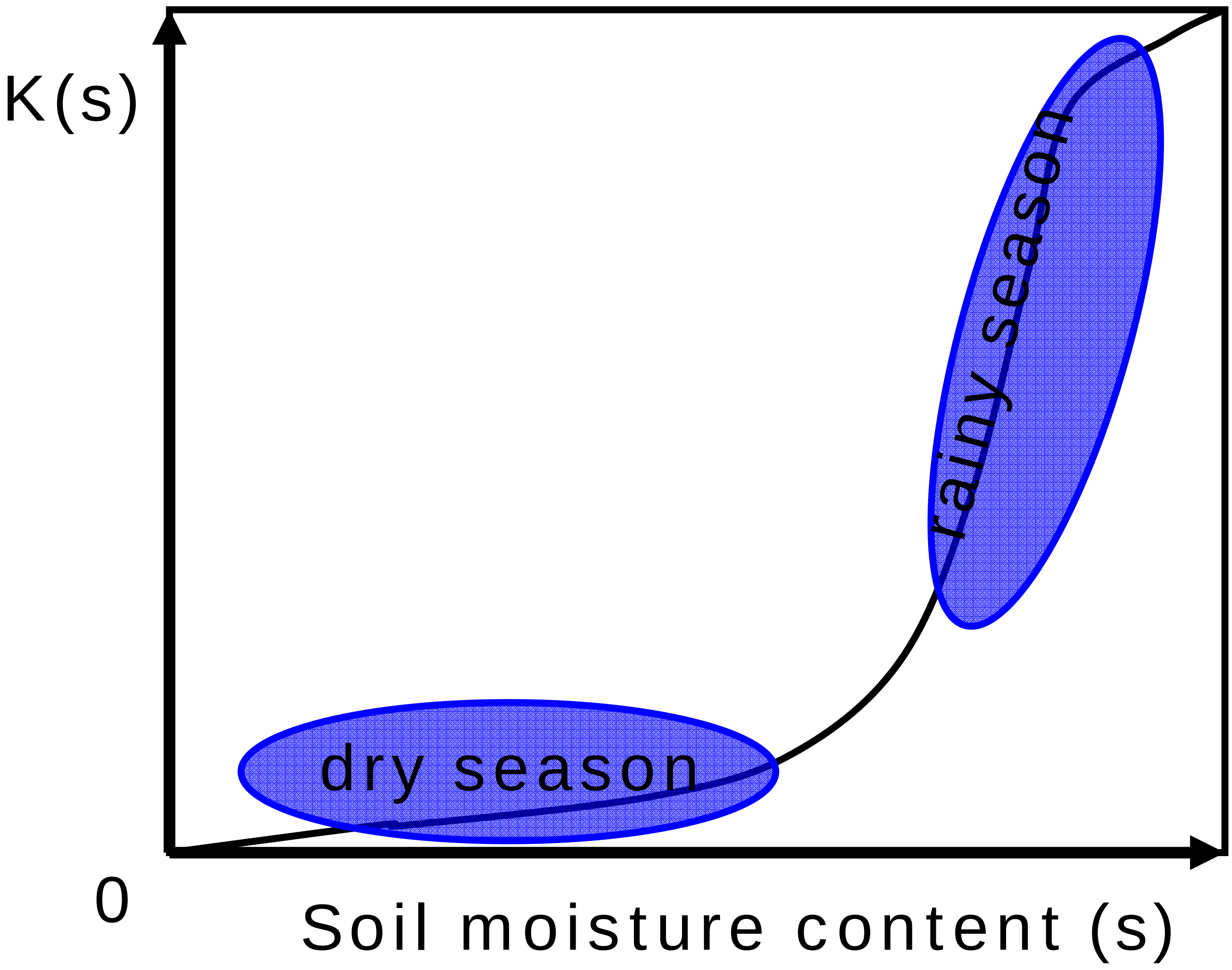
3.2 the value of additional data

Flux data as additional data

Hutley et al., (2000, 2005)

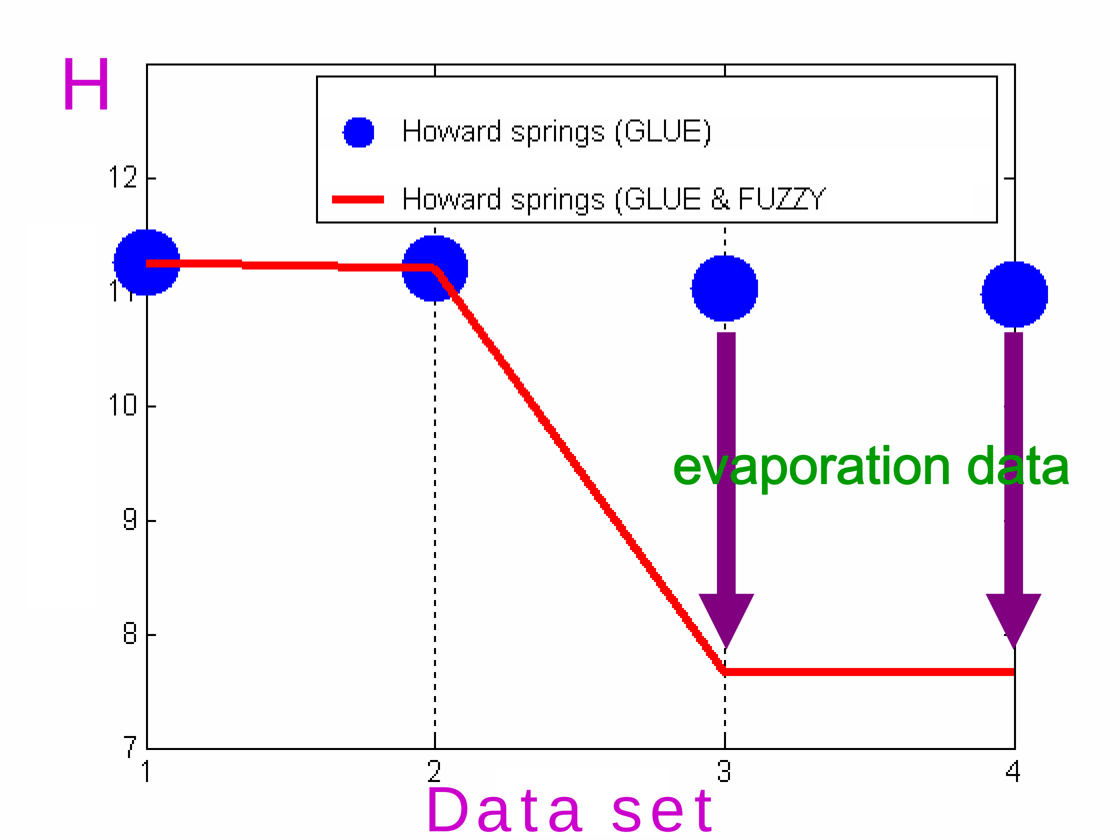


The use of evaporation data (2001/1/1 – 2003/3/29)



Uncertainty (H): measure & reduction

$$H = -\Sigma L_i \log_2 L_i$$ Shannon entropy measure (1948a, b)



4. Summary

- Regarding uncertainty in streamflow prediction

1. Uncertainty analysis using GLUE revealed poor CREW performance at peak flows.
2. The use of flux data helped reduce uncertainties in streamflow prediction which were quantified by Shannon entropy.

4. Summary

- Regarding what we learn from uncertainty analysis

1. Through the simulation of Susannah brook and Howard springs using CREW with GLUE showed that uncertainty bounds of streamflow were related to annual water balances of catchments.

2. At the simulation of Howard springs, low flows are sensitive to the changes in evaporation process at the beginning of the rainy season, but insensitive at the end of rainy season due to the nonlinear control of soil with respect to water movement.

Thank you!!!